Turbulente Strömungen
WS 2015/2016
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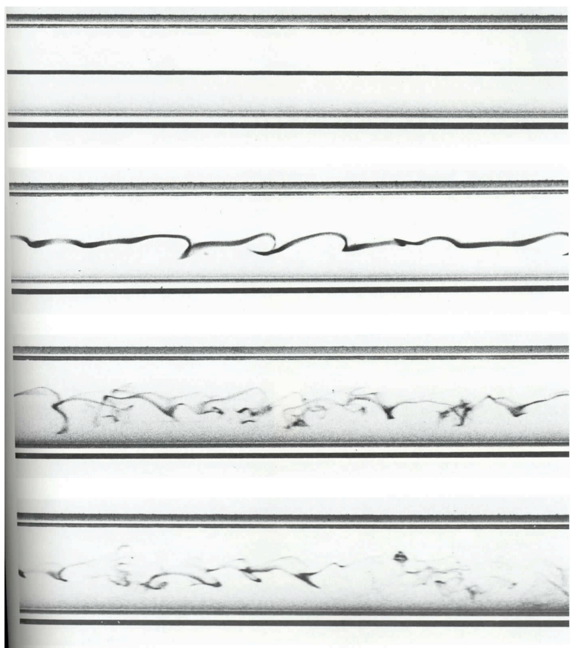
Handout #2

Handout # 2: Introduction to Turbulence

What is turbulence and why is it important?

Reynolds' Classical Experiment



103. Repetition of Reynolds' dye experiment. Osborne Reynolds' celebrated 1883 investigation of stability of flow in a tube was documented by sketches rather than photography. However the original apparatus has survived at the University of Manchester. Using it a century later, N. H. Johannesen and C. Lowe have taken this sequence of photographs. In laminar flow a filament of colored water introduced at a bell-shaped entry extends undisturbed the whole length of the glass tube. Transition is seen in the second of the photographs as the speed is increased; and the last two photographs show fully turbulent flow. Modern traffic in the streets of Manchester made the critical Reynolds number lower than the value 13,000 found by Reynolds.

Observation:

1. For sufficiently large Re number, flow becomes turbulent
(flow develops instability when it cannot be suppressed by viscosity anymore)

Governing Equations

Navier-Stokes equations describe turbulence, but the solution of realistic problems is too expensive

Momentum Equation
$$\frac{\partial U_j}{\partial t} + U_i \frac{\partial U_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial P}{\partial x_j} + \nu \frac{\partial^2 U_j}{\partial x_i^2}$$

Consider jet and non-dimensionalize with jet diameter D and jet exit bulk velocity U_b

$$\hat{U}_j = \frac{U_j}{U_b} \qquad \hat{x}_j = \frac{x_j}{D} \qquad \hat{p} = \frac{p}{\rho U_b^2} \qquad \hat{t} = \frac{tU_b}{D}$$

$$\Rightarrow \quad \frac{\partial \hat{U}_j}{\partial \hat{t}} + \hat{U}_i \frac{\partial \hat{U}_j}{\partial \hat{x}_i} = -\frac{\partial \hat{P}}{\partial \hat{x}_j} + \underbrace{\frac{\nu}{U_b D}}_{1/\mathrm{Re}} \frac{\partial^2 \hat{U}_j}{\partial \hat{x}_i^2}$$

For turbulent flows, viscous term important even if Reynolds number becomes very large.

Similarity

Unknowns: $\hat{U}_j, \hat{p} = f(\hat{x}_j, \hat{t}, U_b, D, \nu)$

Buckingham Π-Theorem:

#Nondimensional Groups = #Parameters - #Units

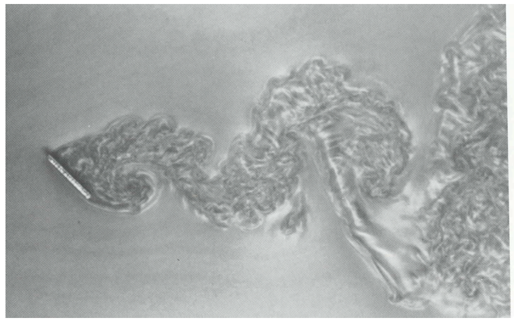
= 7 - 2 = 5

$\Rightarrow$ Five nondimensional groups: $\hat{x}_j, \hat{t}, \mathrm{Re}$

For large Re, solution becomes independent of Re

Example: Wake of a grounded ship at Re = 107 and a wake at Re = 4300

173. Wake of a grounded tankship. The tanker *Argo Merchant* went aground on the Nantucket shoals in 1976. Leaking crude oil shows that she happened to be inclined at about 45° to the current. Although the Reynolds number is approximately 10^7, the wake pattern is remarkably similar to that in the photograph at the top of the page. *NASA photograph, courtesy of* O. M. Griffin, *Naval Research Laboratory.*



172. Wake of an inclined flat pla
wake behind a plate at 45° angle of a
turbulent at a Reynolds number o
Aluminum flakes suspended in wate
its characteristic sinuous form. C
1981. Reproduced, with permission, f
Annual Review of Fluid Mechanics,
13. © *1981 by Annual Reviews Inc.*

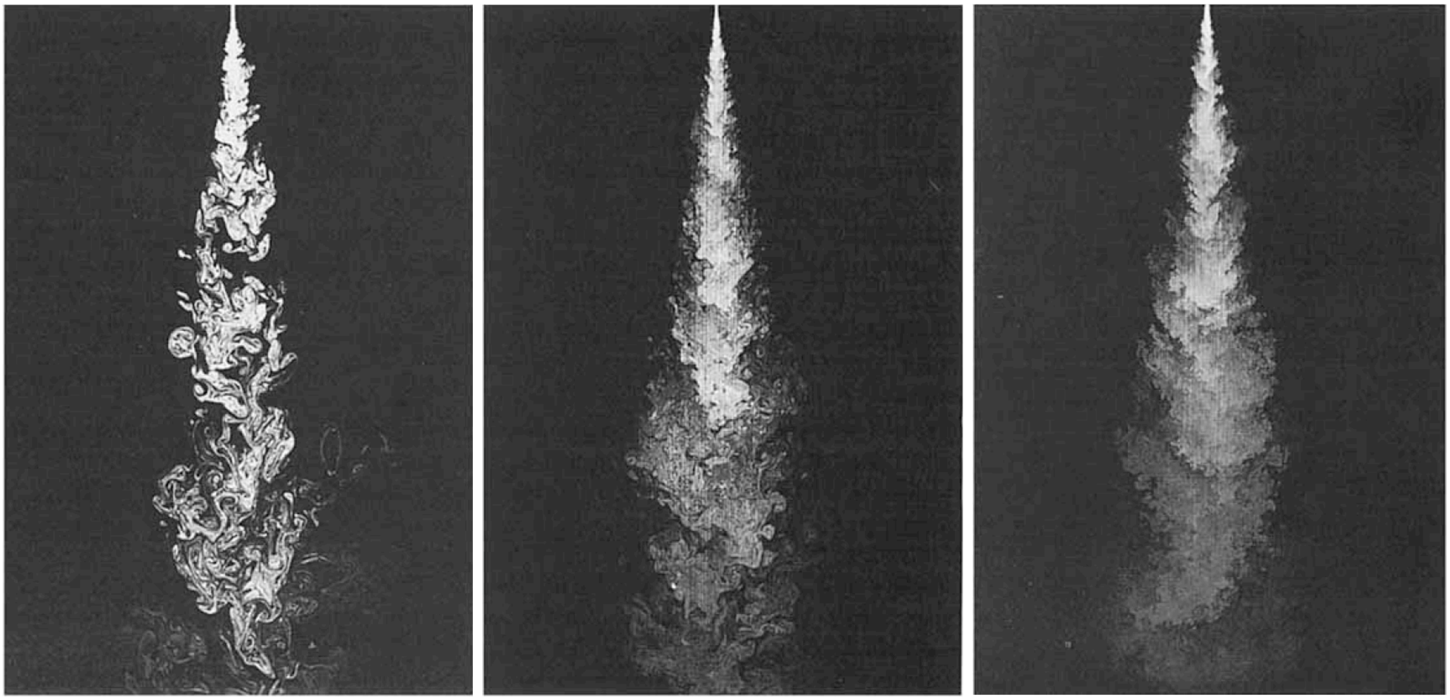
Turbulent Scales

Identify turbulent length scale as the characteristic size of a pocket of fluid of coherent motion

Then, the flow velocity of that pocket relative to the surroundings is the characteristic velocity scale at that length scale

Associated with this velocity scale is the kinetic energy of that scale

Conserved Scalar Turbulent Jets from Dahm and Dimotakis (1990):



Re = 1,500 | Re = 5,000 | Re = 20,000

Observation:

2. Turbulent flows have structures of different length scales

3. Smaller scales depend on Re

4. Large-scale features independent of Reynolds number

Kinetic energy of large scale turbulent motions extracted from mean shear

Turbulence produced at large scales

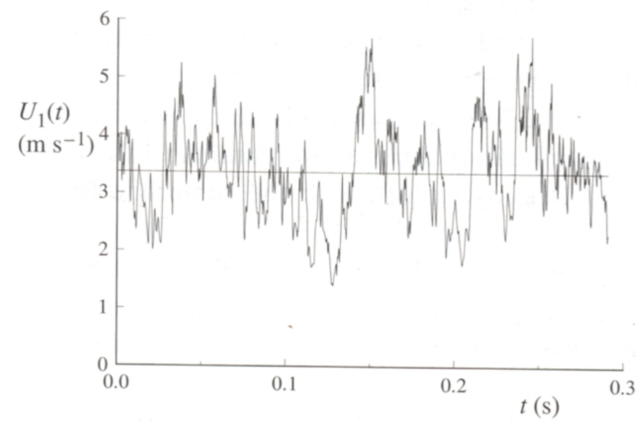


Fig. 1.3. The time history of the axial component of velocity $U_1(t)$ on the centerline of a turbulent jet. From the experiment of Tong and Warhaft (1995).

Observation:

4. Smaller scales lead to larger gradients

$\Rightarrow$ Viscous term becomes important

A Few Conclusions for Turbulent Flows:

- Momentum equation

$$\frac{\partial U_j}{\partial t} + U_i \frac{\partial U_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial P}{\partial x_j} + \nu \frac{\partial^2 U_j}{\partial x_i^2}$$

- For large Re number laminar flow, viscous term disappears
- No damping of disturbances
- Instabilities lead to turbulent flow
- Turbulence produced at large scales representing geometry
- Turbulence generates smaller and smaller scales with larger and larger gradients
- $\partial^2 U_j/\partial x_i^2$ becomes larger until it balances small 1/Re
- Dissipation of turbulence by viscous forces at smallest scales

$\Rightarrow$ Energy transfer from large to small scales

Molecular and Turbulent Mixing

Heat transfer in a room without flow:

$$\frac{\partial \Theta}{\partial t} = \gamma \frac{\partial^2 \Theta}{\partial x_i^2}$$

Scaling with temperature difference across the room $\Delta\Theta$ and room size L

$$\frac{\Delta\Theta}{t_m} \sim \gamma \frac{\Delta\Theta}{L^2}$$

$$\Rightarrow \quad \text{Mixing time } t_m \sim \frac{L^2}{\gamma}$$

Turbulent mixing with turbulent scales proportional to L, U

$$t_t \sim \frac{L}{U}$$

$\Rightarrow$ Ratio of turbulent to molecular mixing time

$$\frac{t_t}{t_m} = \frac{L}{U}\frac{\gamma}{L^2} \sim \frac{1}{\mathrm{Re}} \qquad (\text{for } \mathrm{Pr} = 1)$$

$\Rightarrow$ Re can be interpreted as ratio of molecular to turbulent time scales

$\Rightarrow$ Time to mix over a certain distance is therefore smaller by a factor Re compared with laminar mixing

Example: For a turbulent jet, Re is at least Re $\approx$ 100, therefore mixing in a turbulent jet is at least 100 times faster than for laminar jet.